Period 1, May 9, 2025

~~Cross sections~~
~~# 2 FRQ~~
~~IVT MVT EVT~~
~~Related rates shadow problem~~
~~FTC~~
~~Rolles~~
~~Disc and washer method~~

$y = 2e^{2x}$

$$\frac{\sqrt{3}}{4}\int_1^2 \left(2e^{2x}\right)^2 dx$$

$$\frac{\sqrt{3}}{4}\int_1^2 4e^{4x}\, dx$$

$u = 4x$

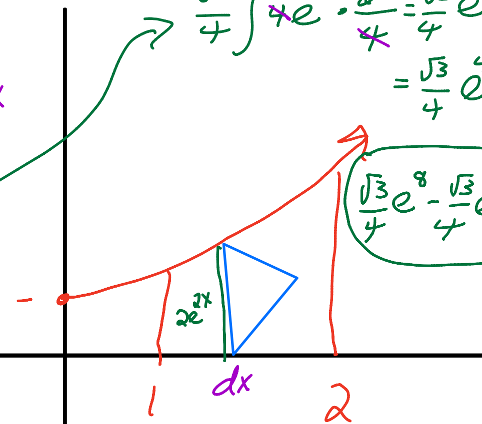
$du = 4dx$

$\frac{du}{4} = dx$

$$\frac{\sqrt{3}}{4}\int \cancel{4}e^{u} \cdot \frac{du}{\cancel{4}} = \frac{\sqrt{3}}{4}e^{u}$$

$$= \frac{\sqrt{3}}{4}e^{4x}\Big]_1^2$$

$$\frac{\sqrt{3}}{4}e^{8} - \frac{\sqrt{3}}{4}e^{4}$$

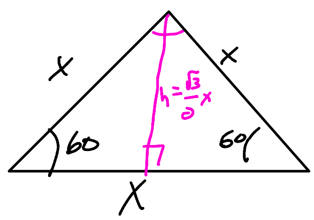


base = x

$\sin 60^\circ = \frac{h}{x}$

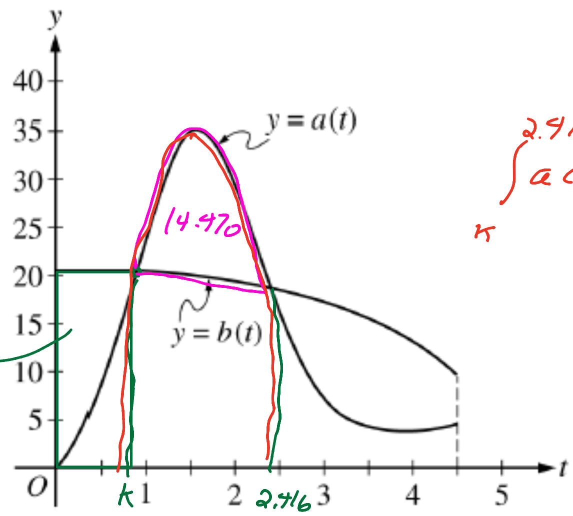
$x \cdot \frac{\sqrt{3}}{2} = \frac{h}{\cancel{x}} \cdot \cancel{x}$

$\frac{\sqrt{3}}{2} \cdot x = h$



$$\text{Area} = \frac{1}{2}x \cdot \frac{\sqrt{3}}{2}x = \frac{\sqrt{3}}{4}x^2 = \frac{\sqrt{3}}{4}(\text{side})^2$$

A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.



$(20.5)(0.89204)$

$\int_k^{2.416} a(T)\,dT - 14.470 = \int_k^{2.416} b(T)\,dt$

During the time interval $0 \leq t \leq 4.5$ hours, water flows into tank *A* at a rate of $a(t) = (2t - 5) + 5e^{2\sin t}$ liters per hour. During the same time interval, water flows into tank *B* at a rate of $b(t)$ liters per hour. Both tanks are empty at time $t = 0$. The graphs of $y = a(t)$ and $y = b(t)$, shown in the figure above, intersect at $t = k$ and $t = 2.416$.

(a) How much water will be in tank *A* at time $t = 4.5$?

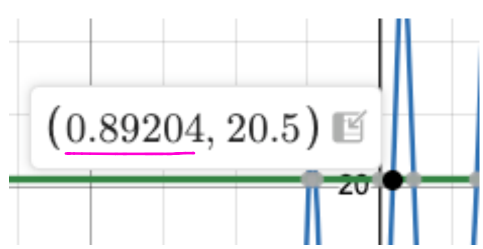
$$\int_0^{4.5} \left[(2T-5) + 5e^{2\sin T} \right] dT$$

$$\int_0^{4.5} \left[(2t-5) + 5e^{2\sin t} \right] dt = 66.5321282934$$

66.532 Liters

$(2T-5)+5e^{2\sin T} = 20.5$

$k = 0.89204$



(b) During the time interval $0 \le t \le k$ hours, water flows into tank B at a constant rate of 20.5 liters per hour. What is the difference between the amount of water in tank A and the amount of water in tank B at time $t = k$?

amount in Tank A $= \int_0^k [(2T-5)+5e^{2\sin T}]\, dT$

amount in Tank b $= \int_0^k 20.5\, dT = 20.5T\Big]_0^k = 20.5 \cdot k - \cancel{20.5 \cdot 0}$

$$\left[\int_0^k [(2T-5)+5e^{2\sin T}]\, dT\right] - 20.5k$$

$$\int_0^{0.89204} \left[(2t-5)+5e^{2\sin t}\right] dt$$

$= 7.68762870238$

$7.68762870238 - (20.5)(0.89204)$

$- 18.28682$

$= -10.599193$

difference

10.599

(c) The area of the region bounded by the graphs of $y = a(t)$ and $y = b(t)$ for $k \le t \le 2.416$ is 14.470.

How much water is in tank B at time $t = 2.416$?

$$\int_{0.89204}^{2.416} \left[(2t-5)+5e^{2\sin t}\right] dt$$

$= 44.497056407$

Tank B From k To 2.416

$$\left[\int_k^{2.416} a(T)\, dT\right] - 14.470 = \int_k^{2.416} b(T)\, dT$$

$44.497056407 - 14.470 = 30.0270564) + 20.5(0.89204)$

$= 48.3138$ Liters in Tank b

(d) During the time interval $2.7 \leq t \leq 4.5$ hours, the rate at which water flows into tank *B* is modeled by $w(t) = 21 - \frac{30t}{(t-8)^2}$ liters per hour. Is the difference $w(t) - a(t)$ increasing or decreasing at time $t = 3.5$? Show the work that leads to your answer.

$$21 - \frac{30T}{(T-8)^2} - \left[(2T-5) + 5e^{2\sin T}\right] = \text{difference}$$

$w'(T) - a'(T)$ at $T = 3.5$

+ increasing

− decreasing

$$\frac{d}{dt}\left[(2t-5) + 5e^{2\sin t}\right] = -2.6430274906$$

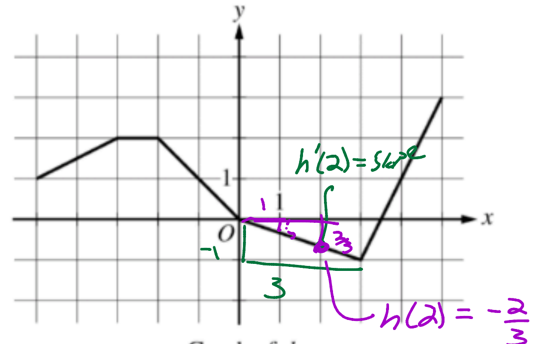
$$\frac{d}{dt}\left[21 - \frac{30t}{(t-8)^2}\right] = -3.78600823045$$

$-3.786 - (-2.643) = \text{negative}$

decreasing

2017 (6d)

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.
also continious

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

$F'(x) = -2\sin 2x + e^{\sin x}(\cos x)$

$F'(\pi) = -2\sin 2\pi + e^{\sin\pi}(\cos\pi)$

$-2 \cdot 0 + e^0(-1) = -1$

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer. MVT

a) $m = -1$ $\quad F(\pi) = \cos 2\pi + e^{\sin\pi} = 1 + e^0 = 1 + 1 = 2$

Point $(\pi, 2)$

$Y - \cancel{2} = -1(x - \pi) + 2$ $\quad$ Line $= -1(x - \pi) + 2$

$+2$

b) $K(x) = h(F(x))$

$K'(x) = h'(F(x)) \cdot F'(x)$

$K'(\pi) = h'(F(\pi)) \cdot F'(\pi)$

$h'(2) \cdot -1$

$-\frac{1}{3} \cdot -1 = \frac{1}{3}$

c) $m(x) = g(-2x) \cdot h(x)$

$m'(x) = g'(-2x) \cdot -2 \cdot h(x) + g(-2x) \cdot h'(x)$

$m'(2) = g'(-4) \cdot -2 \cdot h(2) + g(-4) \cdot h'(2)$

$-1 \cdot -2 \cdot -\frac{2}{3} + 5 \cdot -\frac{1}{3}$

$-\frac{4}{3} + -\frac{5}{3} = -\frac{9}{3} = -3$

$(-5, 10)$

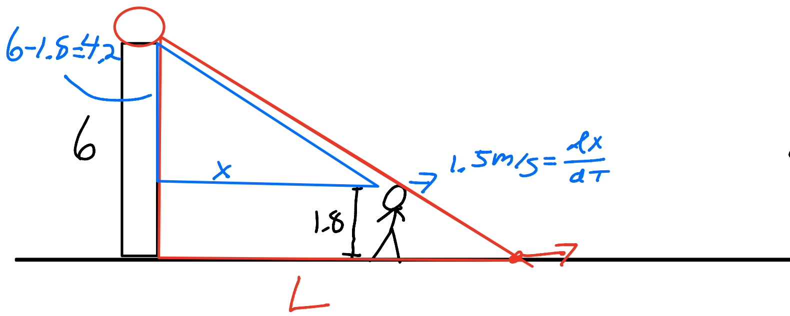
$(-3, 2)$

$\frac{10 - 2}{-5 - (-3)} = \frac{8}{-2} = -4$

g(x) is continious and diff so by MVT

There does exist a slope of -4

A 1.8-meter tall man walks away from a 6.0-meter lamp post at the rate of 1.5 m/s. The light at the top of the post casts a shadow in front of the man. How fast is the "head" of his shadow moving along the ground?



$$\frac{4.2}{x} = \frac{6}{L}$$

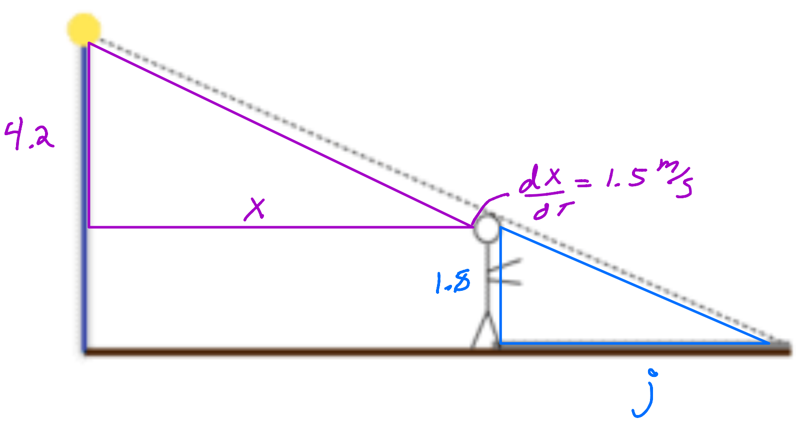
$$4.2L = 6x$$

$$4.2\frac{dL}{dT} = 6\frac{dx}{dT}$$

$$\frac{\cancel{4.2}\frac{dL}{dT}}{\cancel{4.2}} = \frac{6(1.5)}{4.2}$$

$$\frac{dL}{dT} = 2.14286 \, m/s$$

how fast is the shadow length changing



$$\frac{4.2}{x} = \frac{1.8}{j}$$

$$4.2j = 1.8x$$

$$4.2\frac{dj}{dT} = 1.8\frac{dx}{dT}$$

$$\frac{dj}{dT} = \frac{(1.8)(1.5)}{4.2} =$$

$$= 0.64286$$

$2.14286 - 1.5 = 0.64286$

Tip of shadow speed − Person speed = Speed of shadow length change

$$\frac{d}{dT}\left[\int_5^{T^2} x^5 \cos^4 3x\, e^{5x^3}\, dx\right]$$

$x = T^2$

$dx = 2T$

$$(T^2)^5 \cos^4 3(T^2)\, e^{5(T^2)^3} \cdot 2T - 0$$

$$\frac{d}{dT}\left[\int_{T^2}^{5} x^5 \cos^4 3x\, e^{5x^3}\, dx\right] = 0 - (T^2)^5 \cos^4 3(T^2)\, e^{5(T^2)^3} \cdot 2T$$

$$\int_3^6 (5x)^4 \cdot \cos 5x \cdot e^{5x}\, dx = \int_{15}^{30} u^4 \cos u\, e^u \cdot \frac{du}{5}$$

$u = 5x$

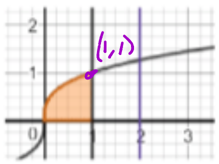
$du = 5\,dx \Rightarrow \frac{du}{5} = dx$

$u = 5 \cdot 3 = 15$

$u = 5 \cdot 6 = 30$

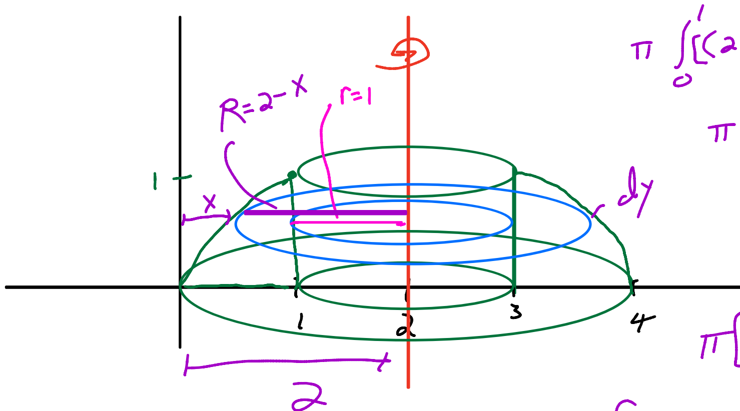
7.

Find the volume of the solid generated by revolving the region bounded by $y = \sqrt[3]{x}$, the x-axis, and the line $x = 1$ about the line $x = 2$.



$y = \sqrt[3]{x}$

$y^3 = x$



$\int_0^1 \pi (R^2 - r^2)\,dy$

$\pi \int_0^1 [(2-\underbrace{x}_{y^3})^2 - 1^2]\,dy$

$\pi \int_0^1 [(2-y^3)^2 - 1]\,dy$

$\pi \int_0^1 [4 - 4y^3 + y^6 - 1]\,dy$

$\pi \int_0^1 [3 - 4y^3 + y^6]\,dy$

$\pi \left[3y - y^4 + \frac{1}{7}y^7\right]\Big|_0^1$

$\pi\left[3(1) - 1^4 + \frac{1}{7}(1)^7\right] - \cancel{\pi\left[3(0) - 0^4 + \frac{1}{7}(0)^7\right]}$

$\pi(3 - 1 + \frac{1}{7})$

$2\frac{1}{7}\pi = \frac{15}{7}\pi$

10. Which of the following definite integrals are equal to $\lim_{n\to\infty}\sum_{k=1}^{n}\left(-1+\frac{4k}{n}\right)^2\frac{4}{n}$ 4

1

Length of interval

I. $\int_{-1}^{3} x^2\,dx$ $(-1)^2=1$ $(3^2)=9$

II. $\int_{0}^{4}(-1+x)^2\,dx$ $(-1+0)^2=1$ $(-1+4)^2=9$

$k=1$ $\lim_{n\to\infty}\left(-1+\frac{4\cdot 1}{n}\right)^2=1$

III. $\int_{0}^{1}\cancel{4(-1+4x)^2}\,dx$ 4 36

$k=n$ $\lim_{n\to\infty}\left(-1+\frac{4n}{n}\right)^2=9$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III only

Theorem 4.80. Derivative of Inverse Functions. *Given an invertible function $f(x)$, the derivative of its inverse function $f^{-1}(x)$ evaluated at $x=a$ is:*

$$[f^{-1}]'(a)=\frac{1}{f'[f^{-1}(a)]}$$

$F(x)=3x^3-2x^2+x-4$

$F^{-1}(x)=g(x)$

Find $g'(14)$

$g(14)=a$

$F(a)=14$

$14=3x^3-2x^2+x-4$

$F(2)=14$

$g(14)=2$

Test

x	y
0	$-4=F(0)$
+1	$-2=F(1)$
+2	$14=24-8+2-4$
+3	
-1	$-10=F(-1)$

$F'(x)=9x^2-4x+1$

$g'(14)=\frac{1}{F'(g(14))}=\frac{1}{F'(2)}=\frac{1}{29}$

$F'(2)=9(2)^2-4(2)+1=36-8+1=29$

disc method

$$\pi \int_a^b R^2 \, dx$$